Standard Curve RX-202A

Standard Curve RX-202A: Measurement Voltage = 1-3 mV DC at T > 1.2 K and at T < 1.2 K we use an AC resistance bridge with excitation of 100 μV ±50% from 0.1 K to 1.2 K and 30 μV ±50% from 0.05 K to 0.1 K

T (K)	Resistance (ohms)	dr/dt (ohms/K)	Sd	T (K)	Resistance (ohms)	dr/dt (ohms/K)	Sd
0.050	69,191.1	-3,186,379	-2.303	1.40	3,819.87	-878.56	-0.322
0.055	56,177.9	-2,081,895	-2.038	1.60	3,664.89	-683.65	-0.298
0.060	47,523.9	-1,475,460	-1.863	1.80	3,542.40	-549.21	-0.279
0.065	40,978.0	-1,147,552	-1.820	2.00	3,442.66	-453.22	-0.263
0.070	35,958.6	-872,400	-1.698	2.20	3,359.46	-382.06	-0.250
0.075	32,137.3	-667,159	-1.557	2.40	3,288.72	-327.55	-0.239
0.080	29,186.0	-522,604	-1.432	2.60	3,227.67	-284.62	-0.229
0.085	26,839.0	-421,726	-1.336	2.80	3,174.32	-250.05	-0.221
0.090	24,920.5	-349,907	-1.264	3.00	3,127.23	-221.72	-0.213
0.095	23,308.1	-297,301	-1.212	3.20	3,085.31	-198.16	-0.206
0.100	21,927.1	-256,913	-1.172	3.40	3,047.72	-178.34	-0.199
0.110	19,665.3	-199,443	-1.116	3.60	3,013.78	-161.50	-0.193
0.120	17,877.8	-160,293	-1.076	3.80	2,982.96	-147.07	-0.187
0.130	16,425.0	-131,540	-1.041	4.00	2,954.82	-134.64	-0.182
0.140	15,224.4	-109,581	-1.008	4.20	2,929.00	-123.74	-0.177
0.150	14,217.3	-92,468	-0.976	4.40	2,905.22	-114.26	-0.173
0.160	13,363.5	-78,813	-0.944	4.60	2,883.19	-106.38	-0.170
0.170	12,632.2	-67,832	-0.913	4.80	2,862.63	-99.178	-0.166
0.180	11,999.9	-58,934	-0.884	5.00	2,843.53	-91.772	-0.161
0.190	11,448.1	-51,667	-0.858	5.50	2,801.46	-78.082	-0.153
0.200	10,962.3	-45,681	-0.833	6.00	2,764.85	-68.564	-0.149
0.220	10,145.8	-36,482	-0.791	7.00	2,704.17	-53.809	-0.139
0.240	9,485.27	-29,925	-0.757	8.00	2,655.64	-43.820	-0.132
0.260	8,937.93	-25,047	-0.729	9.00	2,615.56	-36.741	-0.126
0.280	8,475.70	-21,328	-0.705	10.00	2,581.53	-31.556	-0.122
0.300	8,079.35	-18,420	-0.684	11.00	2,552.04	-27.594	-0.119
0.320	7,735.11	-16,089	-0.666	12.00	2,526.07	-24.465	-0.116
0.340	7,432.95	-14,186	-0.649	13.00	2,502.91	-21.923	-0.114
0.360	7,165.50	-12,609	-0.633	14.00	2,482.08	-19.813	-0.112
0.380	6,926.91	-11,286	-0.619	15.00	2,463.18	-18.029	-0.110
0.400	6,712.76	-10,160	-0.605	16.00	2,445.93	-16.502	-0.108
0.420	6,519.44	-9,195.3	-0.592	17.00	2,430.11	-15.179	-0.106
0.440	6,344.07	-8,362.5	-0.580	18.00	2,415.52	-14.024	-0.105
0.460	6,184.23	-7,636.7	-0.568	19.00	2,402.01	-13.007	-0.103
0.480	6,038.03	-6,996.6	-0.556	20.00	2,389.47	-12.107	-0.101
0.500	5,903.85	-6,434.1	-0.545	21.00	2,377.77	-11.305	-0.100
0.550	5,611.84	-5,306.8	-0.520	22.00	2,366.83	-10.588	-0.098
0.600	5,369.38	-4,408.6	-0.493	23.00	2,356.57	-9.9452	-0.097
0.650	5,166.86	-3,755.5	-0.472	24.00	2,346.92	-9.3650	-0.096
0.700	4,990.61	-3,296.1	-0.462	25.00	2,337.82	-8.8403	-0.095
0.750	4,836.60	-2,875.7	-0.446	26.00	2,329.22	-8.3638	-0.093
0.800	4,701.99	-2,518.4	-0.428	27.00	2,321.08	-7.9301	-0.092
0.850	4,583.74	-2,222.2	-0.412	28.00	2,313.35	-7.5338	-0.091
0.900	4,478.78	-1,985.7	-0.399	29.00	2,306.00	-7.1711	-0.090
0.950	4,384.44	-1,793.1	-0.389	30.00	2,299.00	-6.8384	-0.089
1.000	4,298.96	-1,630.8	-0.379	32.00	2,285.92	-6.2479	-0.087
1.050	4,220.96	-1,492.1	-0.371	34.00	2,273.94	-5.7442	-0.086
1.100	4,149.45	-1,371.2	-0.364	36.00	2,262.90	-5.3089	-0.084
1.150	4,083.60	-1,265.0	-0.356	38.00	2,252.67	-4.9301	-0.083
1.200	4,022.75	-1,170.6	-0.349	40.00	2,243.15	-4.5986	-0.082

Polynomial Representation

Curve RX-202A can be represented by a polynomial equation based on the Chebycheb polynomials which are described below. Three separate ranges are required to accurately describe the curve, with the parameters for these ranges given in Table 1. The polynomials represent Curve RX-202A on the preceding page with RMS deviations on the order of ±0.5 mK below 1 K, ±1 mK below 6 K, ±7 mK below 20 K, and ±30 mK below 40 K.

The Chebycheb equation is of the form:

$$T(x) = \sum_{i=0}^{n} a_i t_i(x) \qquad (1)$$

Where T(x) represents the temperature in kelvin, $t_i(x)$ is a Chebycheb polynomial, and a_i represents the Chebycheb coefficients. The parameter x is a normalized variable given by:

$$x = \frac{(Z - ZL) - (ZU - Z)}{(ZU - ZL)} \qquad (2)$$

where Z is the log (base 10) of the resistance and ZL and ZU designate the log of the lower and upper limit of the resistance over the fit range.

The Chebychev polynomials can be generated from the recursion relation:

$$t_{i+1}(x) = 2x t_i(x) - t_{i-1}(x) \qquad (3)$$

$$t_0 = 1, \quad t_1(x) = x$$

Alternatively, these polynomials are given by:

$$t_1(x) = \cos[i \times \arccos(x)] \qquad (4)$$

The use of Chebycheb polynomials is no more complicated than the use of the regular power series and the offer significant advantages in the actual fitting process. The first step is to transform the measured voltage into the normalized variable using equation 2. Equation 1 is then used in combination with Equations 3 and 4 to calculate the temperature. Programs 1 and 2 provide sample BASIC subroutines which will take the resistance and return the temperature T calculated from Chebychev fits. The subroutines assume the values ZL and ZU have been input along with the degree of the fit. The Chebycheb coefficients are also assumed to be in an array A(0), A(1), …, A(Ndegree).

An interesting property of the Chebychev fit is evident in the form of the Chebycheb polynomial given in Equation 4. No term in Equation 1 will be greater than the absolute value of the coefficient. This property makes it easy to determine the contribution of each term to the temperature calculation and where to truncate the series if the full accuracy is not required.

Table 1. Chebychev fit coefficients.

```
Fit Range:  0.050 K to 0.650 K
Order = 10
A(0) = 0.216272    ZL=3.67248634198
A(1) =-0.297572    ZU=5.08000000000
A(2) = 0.146302
A(3) =-0.083696
A(4) = 0.026669
A(5) =-0.019932
A(6) = 0.003085
A(7) =-0.004804
A(8) = 0.000177
A(9) =-0.001218
A(10)= 0.000286

Fit Range:  0.650 K to 5.0 K
Order = 8
A(0)= 2.129752     ZL=3.44161440913
A(1)=-2.281779     ZU=3.74909980595
A(2)= 0.981996
A(3)=-0.386190
A(4)= 0.143467
A(5)=-0.050844
A(6)= 0.017569
A(7)=-0.006164
A(8)= 0.002311

Fit Range:  5.0 K to 40 K
Order = 6
A(0)= 102.338126   ZL=3.27800000000
A(1)=-161.190611   ZU=3.46671731726
A(2)=  94.158738
A(3)= -43.080048
A(4)=  15.317949
A(5)=  -3.881270
A(6)=   0.540313
```

Program 2. BASIC subroutine for evaluating the the temperature T from the Chebychev series using Equations 1 and 3. An array Tc(Ndegree) must be defined.

```
REM Evaluation of Chebychev series
   x=((Z-ZL)-ZU-Z))/(ZU-ZL)
   Tc(0)=1
   Tc(1)=X
   T=A(0)+A(1)*X
   FOR I=2 to Ndegree
      Tc(I)=2*X*Tc(I-1)-Tc(I-2)
      T=T+A(I)*Tc(I)
   NEXT I
   RETURN
```

Program 2. BASIC subroutine for evaluating the temperature T from the Chebychev series using Equations 1 and 4.

```
REM Evaluation of Chebychev series
   x=((Z-ZL)-ZU-Z))/(ZU-ZL)
   T=0
   FOR I=0 to Ndegree
      T=T+A(I)*COS(I*ARCCOS(X))
   NEXT I
   RETURN
```